Journal of Computational and Applied Mathematics 235 (2011) 4513–4515

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Letter to the editor

Comment on "New types of exact solutions for nonlinear Schrodinger equation with cubic nonlinearity"

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ARTICLE INFO

Article history:
Received 2 December 2010

Keywords:
Nonlinear evolution equation
Nonlinear differential equation
Ordinary differential equation
Common error
Exact solution

ABSTRACT

In this comment we analyze the paper [Abdelhalim Ebaid, S.M. Khaled, New types of exact solutions for nonlinear Schrodinger equation with cubic nonlinearity, J. Comput. Appl. Math. 235 (2011) 1984–1992]. Using the traveling wave, Ebaid and Khaled have found "new types of exact solutions for nonlinear Schrodinger equation with cubic nonlinearity". We demonstrate that the authors studied the well-known nonlinear ordinary differential equation with the well-known general solution. We illustrate that Ebaid and Khaled have looked for some exact solution for the reduction of the nonlinear Schrodinger equation taking the general solution of the same equation into account.

1. Introduction

In paper [1] Ebaid and Khaled considered the nonlinear Schrodinger equation with cubic nonlinearity in the form

$$\mathrm{i}\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+\Omega|\psi|^2\psi=0,\quad \mathrm{i}^2=-1. \tag{1.1}$$

It is known that the Schrodinger equation with a cubic nonlinearity is integrable by the inverse scattering method [2–5]. Multi-soliton, rational and some other solutions are well known.

However to obtain some partial solutions of Eq. (1.1) Ebaid and Khaled use the transformation

$$\psi(x,t)=u(\omega)\mathrm{e}^{\mathrm{i}\alpha t},\quad \omega=\lambda x. \tag{1.2}$$

As a result they obtain the nonlinear ordinary differential equation in the form

$$-\alpha u+\lambda^2u''+\Omega u^3=0,\quad u'=\frac{\mathrm{d}u}{\mathrm{d}\omega}. \tag{1.3}$$

Ebaid and Khaled in [1] have looked for exact solutions of Eq. (1.3) and they have not take into consideration that Eq. (1.3) can be easily integrated once more.

In fact multiplying Eq. (1.3) on u' and integrating it with respect to ω we obtain

$$-C_1-\alpha\frac{u^2}{2}+\lambda^2\frac{(u')^2}{2}+\Omega\frac{u^4}{4}=0. \tag{1.4}$$

Eq. (1.4) can be written in the form

$$(u')^2=-\frac{\Omega}{2\lambda^2}u^4+\frac{\alpha}{\lambda^2}u^2+\frac{2C_1}{\lambda^2}. \tag{1.5}$$

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doi:10.1016/j.cam.2011.02.028

As a result we obtain

$$(u')^2 = Pu^4 + Qu^2 + R \tag{1.6}$$

where

$$P \equiv -\frac{\Omega}{2\lambda^2}, \qquad Q \equiv \frac{\alpha}{\lambda^2}, \qquad R \equiv \frac{2C_1}{\lambda^2}. \tag{1.7}$$

Let us remember that the general solution of Eq. (1.6) is expressed in terms of Jacobi elliptic function. Assuming that r_1, r_2, r_3 and r_4 are roots of the algebraic equation

$$Pu^4 + Qu^2 + R = 0, \tag{1.8}$$

we can rewrite Eq. (1.6) in the form

$$(u')^2 = P(u - r_1)(u - r_2)(u - r_3)(u - r_4). \tag{1.9}$$

Using the following transformation [6,7]

$$u = \frac{r_2(r_1 - r_4)G^2 - r_1(r_2 - r_4)}{(r_1 - r_4)G^2 - (r_2 - r_4)}, \tag{1.10}$$

we obtained

$$(G')^2 = (1 - G^2)(1 - k^2G^2), \tag{1.11}$$

where

$$k^2 = \frac{(r_2 - r_3)(r_1 - r_4)}{(r_1 - r_3)(r_2 - r_4)}. \tag{1.12}$$

General solution of Eq. (1.12) is the well-known Jacobi elliptic function [6–8]

$$G = \mathrm{sn}(P^{1/2}M\omega, k), \tag{1.13}$$

where

$$M^2 = \frac{(r_2 - r_4)(r_1 - r_3)}{4}. \tag{1.14}$$

However, Ebaid and Khaled [1] have searched for the exact solutions of (1.3) using the following ansatz

$$u(\omega) = a_0 + a_{-1}F^{-1}(w) + a_1F(w), \tag{1.15}$$

where a_0, a_1, a_{-1} are the constants to be determined. Function $F(\omega)$ satisfies the equation

$$[F'(\omega)]^2 = PF^4(\omega) + QF^2(\omega) + R. \tag{1.16}$$

Then authors substitute (1.15) in Eq. (1.3) taking into account Eq. (1.16). Setting the coefficients with the same power of function $F(\omega)$ to zero the authors obtain the algebraic system on unknown constants. Solving this system they get an exact solution of Eq. (1.3) in terms of function $F(\omega)$ which satisfy auxiliary Eq. (1.16).

It is easy to see that Eqs. (1.16) and (1.6) are exactly the same. So the application of the auxiliary function method by Ebaid and Khaled we can consider as confusion, because the auxiliary Eq. (1.16) and the reduction of the nonlinear Schrodinger equation with cubic nonlinearity (1.6) coincide.

Let us note in conclusion that Ebaid and Khaled have made one of common errors which was discussed in some recent papers [9–15].

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